

## The Time-of-use Price of Electric Vehicle Charging and Discharging based on Monte Carlo algorithm

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**Abstract:** In order to solve the problem of power balance caused by disorderly charging of electric vehicles (EV) to power grid operation, firstly, taking the electric vehicle in a certain area as the research object, according to the characteristics of charging duration and charging start time, Monte Carlo algorithm is proposed to analyze the daily charging rules of users, and then establish the objective function, simulate and calculate the possible charging characteristics of electric vehicles in various market penetration scales. Secondly, the optimal time-of-use (TOU) price is solved by cuckoo search (CS) algorithm. Finally, Through the example of the actual load of a community, the scheduling strategy can effectively reduce user electricity cost and improve the operation of the power grid.

### 1. Introduction

With the application of electric vehicles and the construction of charging stations, a large number of EV are connected to the power grid as loads. Its battery can be charged and discharged at any time, which increases the pressure of power generation, transmission and distribution systems. If electric vehicles make full use of batteries by charging when the load of the power grid is low, and returning the batteries to the power grid at peak time, it can play the functions of load transfer and load regulation of the power grid<sup>[1-4]</sup>.

The research on charging and discharging behavior has made great progress. Literature [5] used Monte Carlo algorithm to simulate the extraction of initial charging state and initial charging time, and then calculates the charging load curve, in which the assumption that initial charging time obeys uniform distribution is put forward<sup>[5]</sup>. Literature [6] has studied the impact of natural recharge and orderly charge-discharge strategy on the influence of the power load, by means of the orderly charging and discharging management of EV, it can reduce the fluctuation of power grid, but does not take into account the user benefits<sup>[6]</sup>.

According to the actual characteristics of charging users, including the start time and duration of charging, the Monte Carlo algorithm is used to analyze the charging behavior of users, and find charging rules, include the density of charging duration and the start time of charging. Basis on the relationship between discharge quantity and price change, we guide users behavior to avoid the charge discharge, peak load power, then the optimization model is established to reduce the fluctuation of power grid and users cost. Finally, A new heuristic algorithm named "cuckoo search algorithm" is used to obtain the peak and valley TOU price and the user's charging and discharging plan. The validity of the TOU price is verified by an example, and the optimization of order charging and discharging is realized.

## 2. Charging Behavior Analysis of Electric Vehicles

Electric vehicle charging has great randomness<sup>[7]</sup>. Monte Carlo analysis method, is a calculation method which uses random sampling statistics to estimate mathematical functions. The charging start time and charging duration characteristics of electric vehicle users are analyzed by Monte Carlo algorithm<sup>[8]</sup>. The implementation steps are as follows:

(1) Established the objective function of the model for predicting the charge and discharge behavior of EV. It divides a day into 24 periods in terms of hours. The total charging capacity of the  $i$  period can be expressed as follows:

$$P_i = \sum_{j=1}^n p_{ij} \quad (1)$$

Where:  $p_{ij}$  is the charging and discharging capacity of the  $NO.j$  EV in the  $i$  period;  $n$  means that in the  $i$  period, a total of  $NO.n$  EV exchange power with the power grid.

(2) Assuming that the battery is full before driving, the constraints of the calculation model for predicting the charge and discharge behavior of EV can be formulated as follows:

$$\Delta T_{ij} = \frac{(1 - S_{ij})Q_i}{P_{ij}} \quad (2)$$

$$t_0 \in (T_{0i}, T_{0i} + \Delta T_{ij}) \quad (3)$$

Where:  $\Delta T_{ij}$  is the maximum charging duration of the  $NO.j$  EV in  $i$  period,  $T_{0i}$  is the starting time of the charging period in  $i$  period;  $S_{ij}$  is the initial battery capacity state of the  $NO.j$  EV in  $i$  period, which is related to the daily travel mileage;  $t_0$  is the starting charge time of each EV under full conditions. It is generated randomly in the range of  $[T_{0i}, T_{0i} + \Delta T_{ij}]$ .

(3) Convergence analysis

$$\beta_i = \frac{\sqrt{V_i(\bar{p})}}{\bar{p}_i} = \frac{\sigma_i(\bar{p})}{\sqrt{k} \bar{p}_i} \quad (4)$$

Where:  $\beta_i$  is the variance coefficient of charging load at  $NO.i$  time,  $V_i(\bar{p})$  is the variance of charging and discharging capacity at  $NO.i$  time,  $\bar{p}_i$  is the expected value of charging and discharging capacity at  $NO.i$  time,  $k$  is the calculation times, and  $\sigma_i(\bar{p})$  represents the standard deviation of charging and discharging capacity at  $NO.i$  time.

The maximum  $\beta = \max(\beta_i)$  of the coefficient of variance in each period is used as the criterion. We should randomly selected users in this area, and then set the Monte Carlo method to simulate at least 100 times, which requires the coefficient of variance to be at least less than 0.05%. According to the above steps, the analysis was conducted. The results were shown in figure 1:

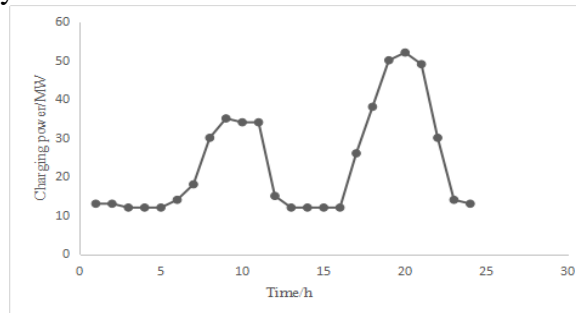


Fig.1 Clustering of charging characteristics of electric vehicles

From Fig 1, it can be found that the daily charging load of users is concentrated between 7:00-12:00 and 18:00-23:00, and the charging start time is concentrated at the time from work to company and from work to home. The average charging time is about 3 hours. Besides, the charging load of electric vehicles increased obviously during the late peak period after 18:00.

### 3. Charging and Discharging Scheduling Model Based on TOU Price

#### 3.1 The Relation between Time-Shared Price and User

According to the daily real-time load of the region and combined with the actual demand of users, TOU divides the electricity time of the whole day into peak, flat and valley three stages, and set different electricity price, which is usually divided into: peak time(07:00 - 11:00, 19:00 - 23:00), the flat section(11:00 - 19:00), valley time(23:00 - the next day 07:00).

In addition, according to the theory of demand response, it is known that the change of electricity price will cause user's electricity demand change. At the same time, the change of user's electricity consumption will be caused by the difference of electricity price in each period of a day. It is expressed by cross elasticity coefficient<sup>[9]</sup>.

$$e_{ij} = \frac{\partial q_i / q_i}{\partial p_j / p_j} \quad (5)$$

Where:  $e_{ij}$  is the elasticity of electricity at  $NO.i$  time to electricity price at  $NO.j$  time, where  $\partial q$  and  $\partial p$  represent the increment of electricity quantity and electricity price. Then at  $I-T$  time, the relationship between electricity quantity and electricity price can be expressed as follows:

$$\begin{bmatrix} \partial q_1 / q_1 \\ \partial q_2 / q_2 \\ \vdots \\ \partial q_t / q_t \end{bmatrix} = E \begin{bmatrix} \partial p_1 / p_1 \\ \partial p_2 / p_2 \\ \vdots \\ \partial p_t / p_t \end{bmatrix} \quad (6)$$

$$E = \begin{bmatrix} e_{11} & e_{12} & \cdots & e_{1t} \\ e_{21} & e_{22} & \cdots & e_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ e_{t1} & e_{t2} & \cdots & e_{tt} \end{bmatrix} \quad (7)$$

Among them,  $E$  represents the elastic coefficient matrix of electricity consumption and electricity price.

In order to control the charging and discharging of EV in an orderly way, different electricity prices can be used to stimulate the enthusiasm of users to participate. To a certain extent, the charging and discharging of EV can be effectively guided, which will reduce its impact on the operation of the power system, save charging costs and effectively improve the operation efficiency of the power grid<sup>[10-11]</sup>.

#### 3.2 Objective function

In order to guide users to charge and discharge orderly, we divide 24 hours of a day into  $T$  time periods. It is generally believed that the electric vehicle that starts charging in  $i \sim i+1$  period starts charging at  $i$  period, then the total number of charges in  $i \sim i+1$  period is:

$$N_i = N \sum_{j=0}^i \int_j^{j+1} f_c(x) dx \int_{i-j}^{i-j+1} f_s(t) dt \quad (8)$$

Where:  $N$  is the total number of charging vehicles, and the probability density function of  $f_c(x)$  and  $f_s(x)$  is the duration of charging and the start time of charging. It is assumed that the load of the  $T$  period in one area is  $L = [l^1, \dots, l^t, \dots, l^T]$ , which excludes the charge and discharge load of EV. And for each EV,  $Q_{k0} = [q_{k0}^1, \dots, q_{k0}^t, \dots, q_{k0}^T]$  is the charging capacity at different time, which indicates the charging capacity of the  $NO.k$  EV at each time before dispatching. Therefore, the total daily power requirement of this electric vehicle is:

$$E_k = \sum_{t=1}^T q_{k0}^t \quad (9)$$

The charging and discharging power of the user stimulated by price after implementing the TOU rule is shown in the following formula:

$$[q_{kc}^1, q_{kc}^2, \dots, q_{kc}^T]^T = \begin{bmatrix} q_{k0}^1 \\ q_{k0}^2 \\ \vdots \\ q_{k0}^T \end{bmatrix} + \begin{bmatrix} q_{k0}^1 & 0 & \dots & 0 \\ 0 & q_{k0}^2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & q_{k0}^T \end{bmatrix} \bullet E_c \begin{bmatrix} \Delta p_1 / p_1 \\ \Delta p_2 / p_2 \\ \vdots \\ \Delta p_T / p_T \end{bmatrix} \quad (10)$$

$$[q_{kd}^1, q_{kd}^2, \dots, q_{kd}^T]^T = \begin{bmatrix} q_{kd0}^1 \\ q_{kd0}^2 \\ \vdots \\ q_{kd0}^T \end{bmatrix} + \begin{bmatrix} q_{kd0}^1 & 0 & \dots & 0 \\ 0 & q_{kd0}^2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & q_{kd0}^T \end{bmatrix} \bullet E_d \begin{bmatrix} \Delta p_{d1} / p_{d1} \\ \Delta p_{d2} / p_{d2} \\ \vdots \\ \Delta p_{dT} / p_{dT} \end{bmatrix} \quad (11)$$

Where,  $Q_{kc} = [q_{kc}^1, q_{kc}^2, \dots, q_{kc}^T]^T$  and  $Q_{kd} = [q_{kd}^1, q_{kd}^2, \dots, q_{kd}^T]^T$  are the charge and discharge of the  $NO.k$  EV in each period,  $E_c$  and  $E_d$  are the charge and discharge price elasticity coefficient matrix.

After participating in the dispatch, the total power requirement of the  $NO.k$  EV is:

$$E_k' = \sum_{i=1}^T (q_{kc}^i - q_{kd}^i) \quad (12)$$

Thus, the load variance, the peak to valley difference, and the economic benefit of users can be selected as the objective functions that can reflect the stable operation of power grid.

(1) the load variance used to represent the fluctuation of the load, and its smaller values show that the load changes smoothly, the impact of the power grid is small<sup>[12]</sup>. Among them, the charge and discharge capacity of each EV in each period is control variables, that is:

$$\min T_1 = \sum_{i=1}^T \left( l^i + \sum_{k=1}^N (q_{kc}^i - q_{kd}^i) - l_{avr} \right)^2 \quad (13)$$

$$l_{avr} = \sum_{i=1}^T \left( l^i + \sum_{k=1}^N (q_{kc}^i - q_{kd}^i) \right) / T \quad (14)$$

Where:  $l^i$  is the load of original grid during the  $i$  period without charging and discharging, and  $l_{avr}$  is the daily average load of the power grid after implementing the TOU price rulers.

(2) the peak valley of the power grid

$$\min T_2 = \max \left( l^i + \sum_{k=1}^N (q_{kc}^i - q_{kd}^i) \right) - \min \left( l^j + \sum_{k=1}^N (q_{kc}^j - q_{kd}^j) \right) \quad (15)$$

(3) the objective is to minimize the user's electricity cost under the TOU price rulers,

$$\min T_3 = \sum_{i=1}^T \sum_{k=1}^N (q_{kc}^i c_i - q_{kd}^i d_i) \quad (16)$$

Where:  $c_i$  its the electricity price of electric vehicles when they are charged, while  $d_i$  represents the electricity price of users when they are discharged in reverse direction.

The above objective functions are mutually related, and in order to optimize the method, the linear weighted sum method is adopted. That is, single objective optimization is used to instead multi-objective optimization problem<sup>[13]</sup>. Besides, due to the dimension of each objective function is not uniform, the normalization of each function is carried out as follows:

$$\min T = \lambda_1 (T_1 / T_{1\max}) + \lambda_2 (T_2 / T_{2\max}) + \lambda_3 (T_3 / T_{3\max}) \quad (17)$$

Where:  $T_{1\max}$  and  $T_{2\max}$  are the mean variance and peak valley difference of the original grid load curve;  $T_{3\max}$  is the cost of charge in a day when charging in disorder.  $\lambda_1$ ,  $\lambda_2$  and  $\lambda_3$  are the weight coefficients of the objective function, and satisfy  $\lambda_1 + \lambda_2 + \lambda_3 = 1$ .

### 3.3 Constraint conditions

(1) After the implementation of TOU rulers, the change of user's power consumption is as small as possible, that is:

$$\min(\sum_{k=1}^N E_k^i - \sum_{k=1}^N E_k) \quad (18)$$

(2) Battery capacity constraint, that is, the ratio of the remaining capacity to the rated capacity during the charging and discharging process is represented by SOC.

$$SOC_{\min} \leq SOC_{ki} \leq SOC_{\max} \quad (19)$$

Where:  $SOC_{ki}$  is the charge state of the  $NO.k$  electric car at  $NO.i$  time, the  $SOC_{\min}$  is the lower limit of the battery when discharging, and the  $SOC_{\max}$  is the upper limit of charging.

(3) After the implementation of TOU price rulers, the charging cost of users should be less than before, that is:

$$\sum_{i=1}^T (q_{kc}^i c_i - q_{kd}^i d_i) \leq \sum_{i=1}^T q_{k0}^i c_i^0 \quad k \in N \quad (20)$$

Where:  $c_i^0$  is the charging price before the TOU price rulers has been implemented.

## 4. Time-of-use price for electric vehicle charging and discharging

### 4.1 Cuckoo search algorithm

CS is an intelligent optimization algorithm that simulates cuckoo nesting and hatching behavior. It has the advantages of fewer parameters, less influence on convergence speed and strong search ability<sup>[14]</sup>.

In the algorithm, the total number of nests is constant, and the poorer nests are eliminated with a certain probability, and the new nest is reestablished to replace the inferior solution with new and potentially superior solutions.

The CS algorithm updates the bird's nest position in two ways:

(1) Update with Levy Flight Random Search

The location of the  $i$ -th nest in the  $t$ -th generation is  $X_i^{(t)}$ , and the random search path is  $L(\lambda)$ , so the update formula of the nest is:

$$X_i^{(t+1)} = X_i^{(t)} + \alpha \oplus L, \quad i = 1, 2, \dots, n \quad (21)$$

Where: vector  $\alpha$  is the search step control factor, which usually takes a fixed value.  $\oplus$  is the dot product.  $L$  means Levy random search path, that is:

$$L = 0.01 \times \frac{\mu}{|\nu|^{1/\beta}} (X_{best} - X_i^{(t)}) \quad (22)$$

Where: the coefficient 0.01 is the typical flight scale in Levy flight model and  $\beta = 1.5$ ;  $X_{best}$  is the best bird nest at present;  $\mu, \nu$  are normal distribution random numbers, that is:

$$\mu \sim N(0, \sigma_\mu^2), \nu \sim N(0, \sigma_\nu^2) \quad (23)$$

Where:  $\sigma_\mu = [\frac{\Gamma(1+\beta) \sin(\pi\beta/2)}{2^{(1+\beta)/2} \Gamma(1+\beta)/2\beta}]^{1/\beta}$ ,  $\sigma_\nu = 1$ ,  $\Gamma$  is the standard gamma function.

After calculating the undetermined bird's nest by formula (21)~(23), the corresponding objective function value is calculated, and then compared with the old bird's nest. If the quality of the pending nest is better, the nest is updated to  $X_i^{(t+1)} = X_i^{(t)} + \alpha \oplus L$ , otherwise it will remain unchanged, that is  $X_i^{(t+1)} = X_i^{(t)}$ .

(2) Update the bird's nest by fixed frequency  $q$

After each iteration generates a new nest, it generates a comparison between random number  $\varepsilon \in [0,1]$  and probability  $q$ . If  $\varepsilon > q$ , the bird nest  $X_{new}$  is randomly generated, and the fitness value of the objective function is calculated. If the bird's nest is better than the original bird's nest, the bird's nest is updated as follows:

$$X_i^{(t+1)} = X_i^{(t)} + \varepsilon(X_j^{(t+1)} - X_k^{(t+1)}) \quad (24)$$

Where:  $X_j^{(t+1)}$  and  $X_k^{(t+1)}$  are two random solutions, and the updated bird's nest is still remembered as  $X_i^{(t+1)}$ .

## 4.2 Learning and training process

Step 1: initialize the population, randomly generate a set of nest locations, and set relevant parameters, the objective function and maximum number of iterations.

Step 2: For  $r$  group sample data, the objective functions corresponding to each nest are calculated, and select the initial global optimal nest, which is retained to the next generation.

Step3: The nest position is updated by Levy flight to determine whether the constraint function is satisfied or not. If it is satisfied, the objective function is compared and the optimal solution is retained.

Step4: Random number  $\varepsilon$  is generated. The nest location is updated by fixed frequency, and the objective function is calculated. Then take the best bird's nest  $X_{best}$  in this generation.

Step5: determine whether the maximum number of iterations or the target function value  $E(X_{best})$  is expected. If the conditions are met, the study is finished. Otherwise, return to step 2 to recalculate.

## 5. Simulation Analysis

### 5.1 Example data

Taking the actual load of a community as an example, the daily load of the community is shown in table 1. The rated capacity of electric vehicle battery is 30 kW·h, and the charging and discharging power is 3.5 kW, assuming that 10 electric vehicles are involved in the scheduling strategy. The parameters of the cuckoo search algorithm are set as follows: the population size is  $N = 20$ , the maximum iteration number is 100, and the probability  $P=0.25$ .

Table 1 Daily load table of a certain community

Time	1	2	3	4	5	6
P/KW	107.8	90.8	94.8	90	91	100.5
Time	7	8	9	10	11	12
P/KW	163.6	235.8	372.2	415.2	408.7	375
Time	13	14	15	16	17	18
P/KW	326.4	334.2	326.8	346.2	380	440.4
Time	19	20	21	22	23	24
P/KW	480.1	528.9	541	431.7	308.3	195

Peak and valley TOU price are set as follows:

Table 2 Time-of-use price

Time	Charge price(yuan/kW·h)	Discharge price(yuan/kW·h)
Peak time (7:00-11:00 and 19:00-23:00)	1.0133	0.7833
Valley time (0:00-7:00And23:00-24:00)	0.3508	0.2632
ordinary time (11:00-19:00)	0.4880	0.5811

## 5.2 Example results

Figure 2 and figure 3 are respectively for the implementation of TOU price before and after the total load of power grid and EV charging and discharging load. It can be found that the load during peak period is significantly reduced. In addition, the comparison shows that users can discharge the electric vehicles at the peak time of high discharging price and charge them at the low charging price at the valley time. And the optimization result is shown in Table 3.

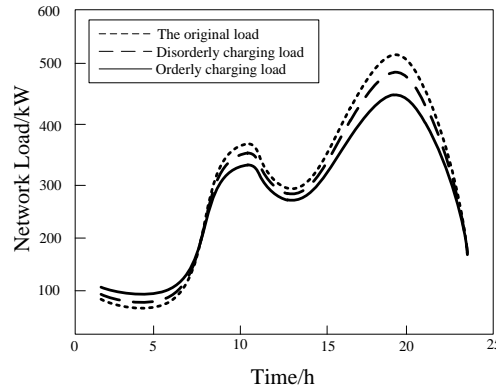


Fig.2 The optimization power grid load curve before and after TOU price

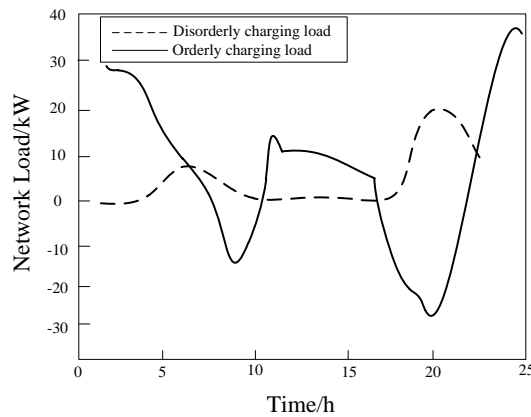


Fig.3 Charging and discharging of electric vehicles before and after TOU price

Table 3 Optimal result of time-of-use price strategy

	Original load of power grid	result of disorderly charging	Ordered charging and discharging results
Peak-valley difference of load/kW	451	475	415
Fluctuation variance of load/(kW) <sup>2</sup>	511582.78	536751.59	445402.78
Cost of Users/yuan	0	58.55	14.03

From the data in table 3, it can be seen that the peak-valley difference and fluctuation variance were increased by 5.32% and 4.92% for the user random charge. But if users can charge and discharge orderly, it can be found that not only decrease the fluctuation of power grid, but also reduces the peak-valley difference and fluctuation variance of power grid load by 7.98% and 12.94%. Moreover, it can significantly reduce the power cost of users. Thus, the goal of orderly charging and discharging is achieved through TOU electricity price.

## 6. Conclusion

The orderly control of charging and discharging of electric vehicles is of great significance to the stable operation of power grid. Taking the TOU price as the background, to improve user engagement charging characteristics and the charging characteristics of the electric vehicle users are analyzed by Monte Carlo algorithm, and the charging law of the users is found. The charging start time and the charging duration are derived. Then, a multi constraint and multi-variable function equation is established to solve the TOU price through the cuckoo search algorithm. Taking a load in an area as an example, the fluctuation rate, peak and valley difference of the power grid and the economic benefits of the user are calculated. Finally, The research shows that this TOU price can reduce the cost of users and the peak load. If it can be connected with smart grid and home micro grid in the future, it will be able to improve the efficiency of electric vehicles and better ensure the stable operation of the grid.

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